Monotone Frameworks

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Data Flow analysis

Data Flow analysis is a form of program analysis which is widely diffused in compiler construction theory.

- It provides a well defined environment for analysis construction
- Provides solution methods
- Sometimes finding a solution can be quite complex

General setting

We model a program with:

- A CFG
- Each node in the CFG represents a basic block, an instruction, a statement
- Variables and arguments are treated as symbols

General setting

- The arcs represent the execution order of the nodes
- An arc from A to B means that B will be executed after A
- A is the predecessor of B

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General setting

There are special nodes in the CFG:

- Entry point, which has no predecessor, also called the *initial* node
- A return point, also called final node, which has no successors

General setting

Conditional nodes:

- Conditional nodes have two successors (and are called bifurcation points), unlike non conditional nodes which have only one.
- On the other side, we may have nodes with two or more predecessors, and these nodes are called confluence points

General setting

- Each node is associated with a state at the entry, and a state at the exit
- The entry and exit states are linked by the behavior of the content of the node
- The arcs in the CFG link different in and out states

Equations

- Once all the elements of the dataflow analysis have been defined, we can write a set of equations representing the constraints on the CFG
- A solution for this set of equations gives origin to a suitable assignement for the dataflow problem

Equation solution method

- Chaotic iteration is the most simple method for computing a solution to the set of equations derived from the dataflow formulation
- Basically, we continue propagating the information until the results from two consecutive iterations do not differ from one another
- It is very hand for manual use, but its complexity it's too high for use in a real-world scenario
- We will see more optimized solution methods later on

Representing state sets

We can represent the sets representing the state of an analysis (e.g., liveness analysis) as:

- Bit vector: Costant time operations, suitable for dense representations
- Standard set: Operations complexity are proportional to the dimension of sets, suitable for sparse representations

Optimization anatomy

- Perform a dataflow analysis and gather information on the program
- Perform some transformations to the program on the basis of the information gathered
- Re-run analysis that may have been invalidated

Some analyses

- Available expressions analysis
- Reaching definitions analysis
- Very busy expressions analysis
- Live variables analysis
- Constant folding and propagation analysis

Available expressions analysis

Goal:

- Find the expressions that have been already computed, and not modified, on all the paths to a certain program point

Common use:

- Avoid the recomputation of expressions that are ready for use in a certain program point

Reaching definitions analysis

Goal:

- Find the assignements which may have been made and not overwritten, when program execution reaches this point along some path

Common use:

- Constructing direct links between blocks that produce values and blocks that use them

Very busy expression analysis

Goal:

- Find the expressions that can be considered *very busy* at the exit point from each program point

Common use:

- Pre-compute the value of the expression at the exit of the program point, and store it for later used (also called *hoisting*

Live variables

Goal:

- Find the variables which *may* be considered live at the exit of each point of the program

Common use:

- Used as the basis for *dead code elimination*

Constant folding and propagation

Goal:

- Compute constant operands of expressions at compile time

Common use:

- Replace uses of constant expressions directly with the constant value

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The even-odd analysis

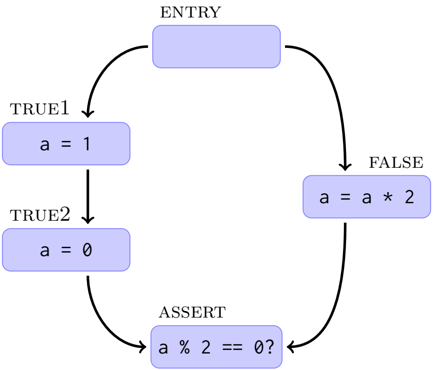
```
void myfunction(unsigned a) {
  if (a > 10) {
    a = 1;
    a = 0;
  } else {
    a = a * 2;
  }

  assert(a % 2 == 0);
}
```

The even-odd analysis

We want to prove that a is even
so that we can drop the assertion.

The even-odd analysis



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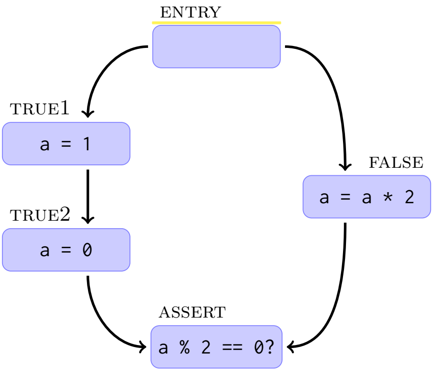

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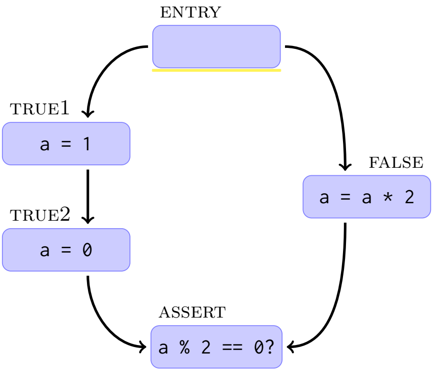

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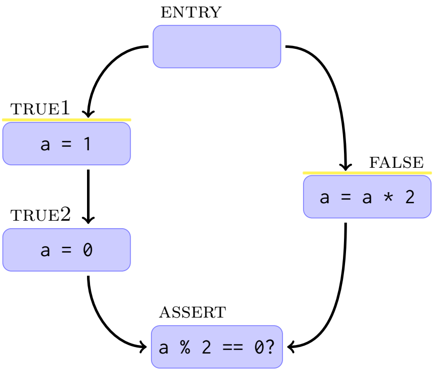

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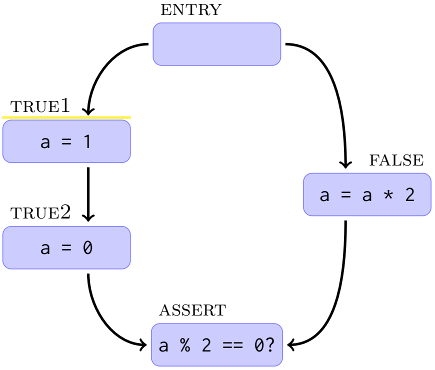

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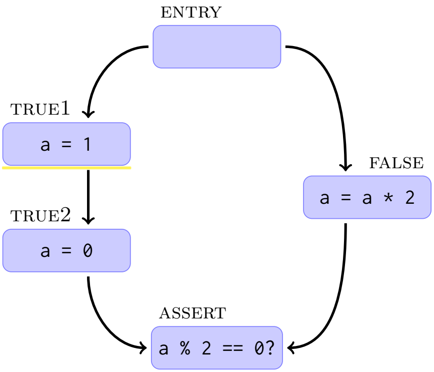

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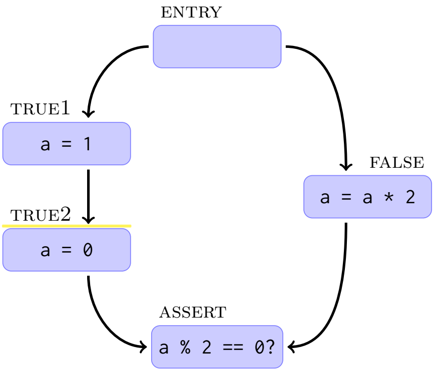

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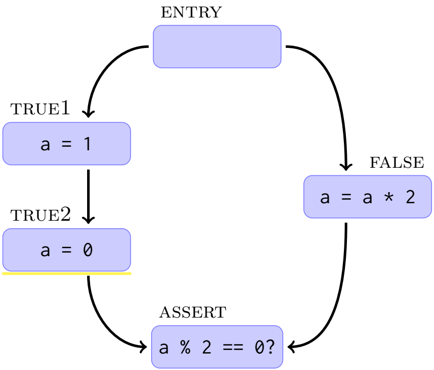

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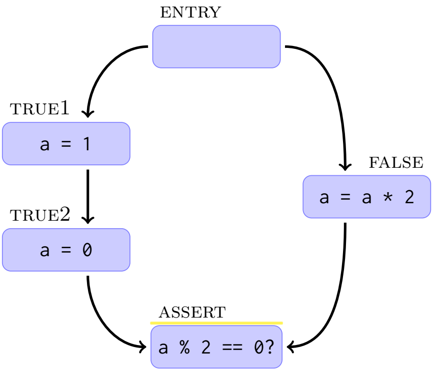

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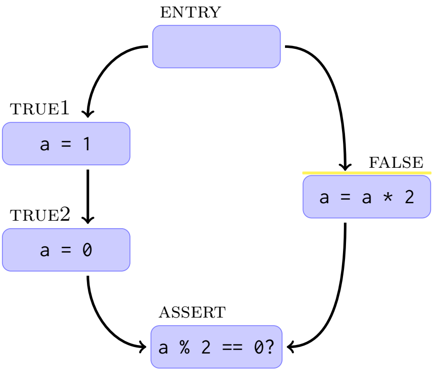

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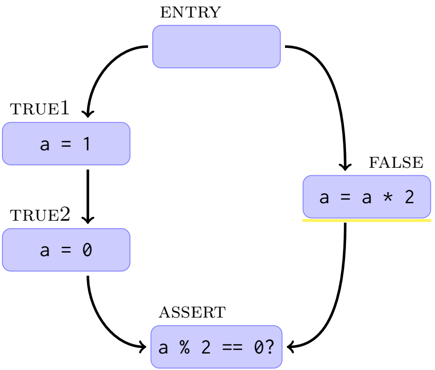

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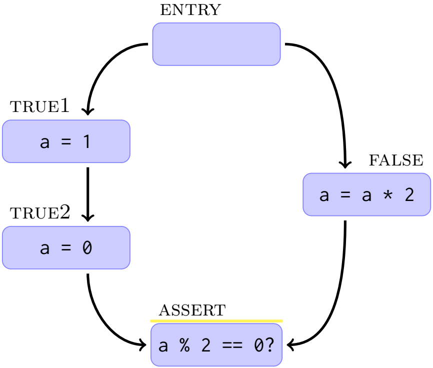

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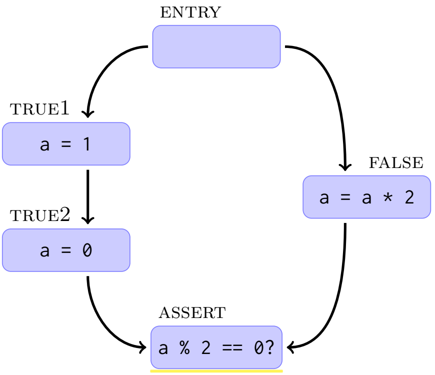

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\text{FALSE}_\bullet & \leftarrow & \text{EVEN} \\
\text{ASSERT}_\bullet & \leftarrow & \text{EVEN}
\end{array}
$$

Let's name things

$\text{ENTRY}, \text{TRUE1}, \text{TRUE2}, \text{FALSE}$	Labels
ENTRY	Extremal label
$\text{ODD}, \text{EVEN}, \text{UNK}$	Possible values
$\text{LABEL}_{\circ}$	Value at the start of LABEL
$\text{LABEL}_{\bullet}$	Value at the end of LABEL
$\text{UNK} \xrightarrow{\times 2} \text{EVEN}$ $\text{ODD} \xrightarrow{+1} \text{EVEN}, \text{EVEN} \xrightarrow{+1} \text{ODD}$	A *transfer function* f_{LABEL}
UNK in $\text{ENTRY}_{\circ}$	Extremal value (i)
$\text{ENTRY} \to \text{TRUE1}$ $\text{TRUE1} \to \text{TRUE2}$	Flow
$\text{ODD} \circ \text{ODD} = \text{ODD}$ $\text{ODD} \circ \text{EVEN} = \text{UNK}$	Combine operator ($\bigsqcup$)

Equation system

We can reframe the problem as a system of equations:

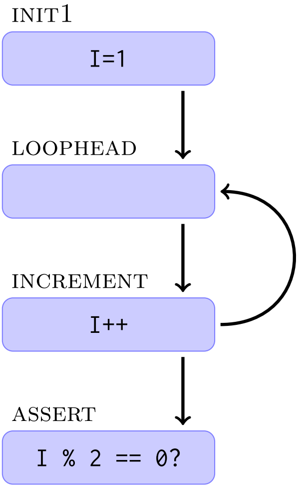
$$
\begin{aligned}
\mathrm{ENTRY}_\circ &= i \\
\mathrm{ENTRY}_\bullet &= f_{\mathrm{ENTRY}}(\mathrm{ENTRY}_\circ) \\
\mathrm{TRUE1}_\circ &= \mathrm{ENTRY}_\bullet \\
\mathrm{TRUE1}_\bullet &= f_{\mathrm{TRUE1}}(\mathrm{TRUE1}_\circ) \\
\mathrm{TRUE2}_\circ &= \mathrm{TRUE1}_\bullet \\
\mathrm{TRUE2}_\bullet &= f_{\mathrm{TRUE2}}(\mathrm{TRUE2}_\circ) \\
\mathrm{FALSE}_\circ &= \mathrm{ENTRY}_\bullet \\
\mathrm{FALSE}_\bullet &= f_{\mathrm{FALSE}}(\mathrm{FALSE}_\circ) \\
\mathrm{ASSERT}_\circ &= \mathrm{TRUE2}_\bullet \sqcup \mathrm{FALSE}_\bullet \\
\mathrm{ASSERT}_\bullet &= f_{\mathrm{ASSERT}}(\mathrm{ASSERT}_\circ)
\end{aligned}
$$

Another example

```
void myfunction(unsigned n) {
  unsigned I = 1;
  while (n-- > 0) {
    I++;
  }

  assert(I % 2 == 0);
}
```

Another example



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$$\text{INCREMENT}_\circ \leftarrow \text{ODD}$$
$$\text{INCREMENT}_\bullet \leftarrow \text{EVEN}$$
$$\text{LOOPHEAD}_\circ \leftarrow \text{UNK}$$
$$\text{ASSERT}_\circ \leftarrow \text{EVEN}$$
$$\text{ASSERT}_\bullet \leftarrow \text{EVEN}$$
$$\text{LOOPHEAD}_\bullet \leftarrow \text{UNK}$$
$$\text{INCREMENT}_\circ \leftarrow \text{UNK}$$
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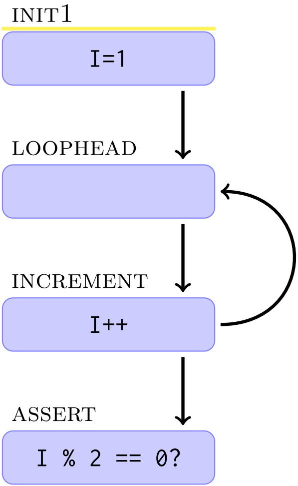
Another example



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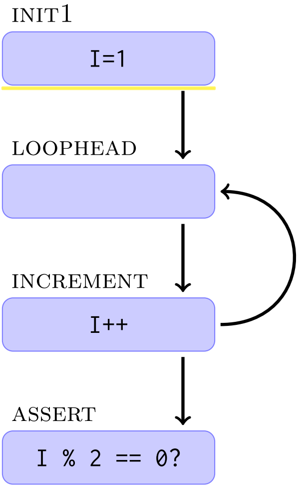

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\text{LOOPHEAD}_\circ &\leftarrow \text{UNK}\\
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$$

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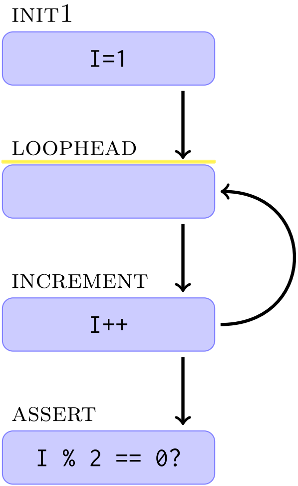

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\text{LOOPHEAD}_\circ &\leftarrow \text{UNK}\\
\text{ASSERT}_\circ &\leftarrow \text{EVEN}\\
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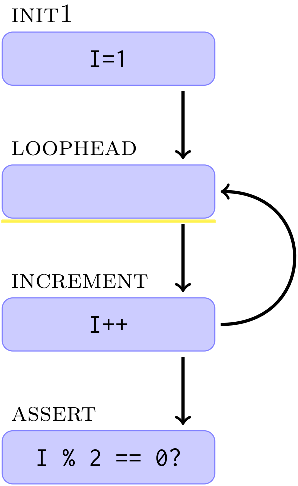

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$\text{ASSERT}_\circ$	$\leftarrow$	EVEN
$\text{ASSERT}_\bullet$	$\leftarrow$	EVEN
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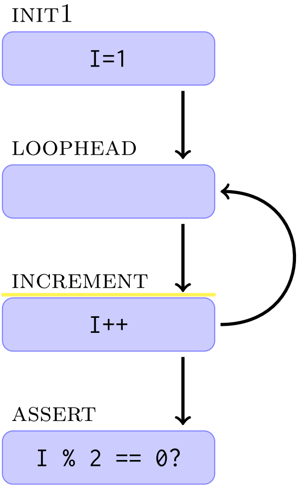

$INIT_\circ \leftarrow UNK$

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$LOOPHEAD_\circ \leftarrow ODD$

$LOOPHEAD_\bullet \leftarrow ODD$

$INCREMENT_\circ \leftarrow ODD$

$INCREMENT_\bullet \leftarrow EVEN$

$LOOPHEAD_\circ \leftarrow UNK$

$ASSERT_\circ \leftarrow EVEN$

$ASSERT_\bullet \leftarrow EVEN$

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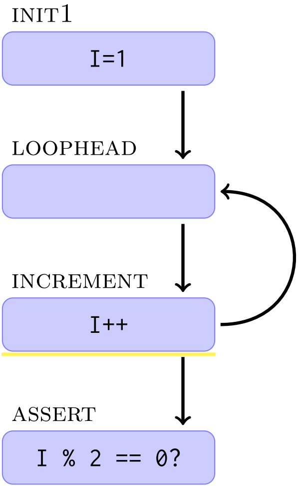

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$\text{LOOPHEAD}_\circ \leftarrow \text{UNK}$

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$\text{LOOPHEAD}_\bullet \leftarrow \text{UNK}$

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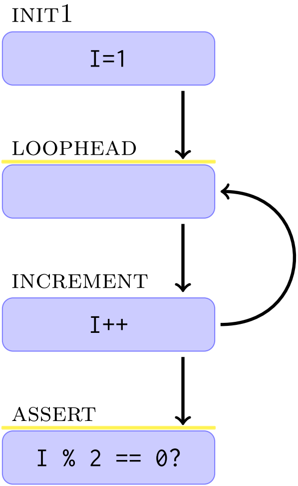

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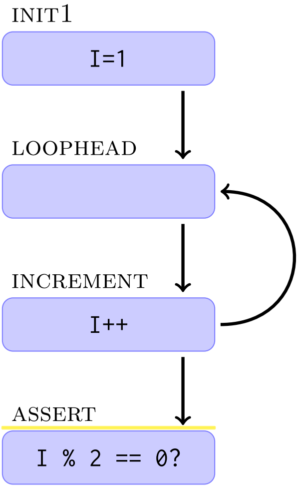
Another example



$\mathrm{INIT}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{INIT}_\bullet \leftarrow \mathrm{ODD}$

$\mathrm{LOOPHEAD}_\circ \leftarrow \mathrm{ODD}$

$\mathrm{LOOPHEAD}_\bullet \leftarrow \mathrm{ODD}$

$\mathrm{INCREMENT}_\circ \leftarrow \mathrm{ODD}$

$\mathrm{INCREMENT}_\bullet \leftarrow \mathrm{EVEN}$

$\mathrm{LOOPHEAD}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{ASSERT}_\circ \leftarrow \mathrm{EVEN}$

$\mathrm{ASSERT}_\bullet \leftarrow \mathrm{EVEN}$

$\mathrm{LOOPHEAD}_\bullet \leftarrow \mathrm{UNK}$

$\mathrm{INCREMENT}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{INCREMENT}_\bullet \leftarrow \mathrm{UNK}$

$\mathrm{ASSERT}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{ASSERT}_\bullet \leftarrow \mathrm{UNK}$

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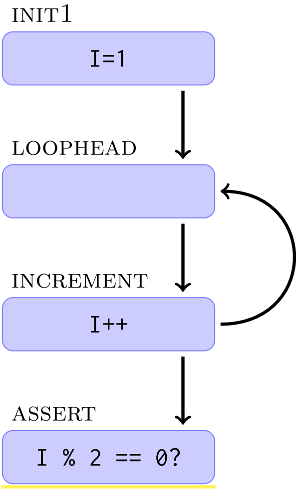

$$
\begin{aligned}
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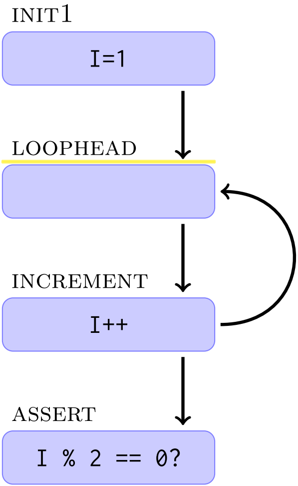
\text{INIT}_\bullet &\leftarrow \text{ODD}\\
\text{LOOPHEAD}_\circ &\leftarrow \text{ODD}\\
\text{LOOPHEAD}_\bullet &\leftarrow \text{ODD}\\
\text{INCREMENT}_\circ &\leftarrow \text{ODD}\\
\text{INCREMENT}_\bullet &\leftarrow \text{EVEN}\\
\text{LOOPHEAD}_\circ &\leftarrow \text{UNK}\\
\text{ASSERT}_\circ &\leftarrow \text{EVEN}\\
\text{ASSERT}_\bullet &\leftarrow \text{EVEN}\\
\text{LOOPHEAD}_\bullet &\leftarrow \text{UNK}\\
\text{INCREMENT}_\circ &\leftarrow \text{UNK}\\
\text{INCREMENT}_\bullet &\leftarrow \text{UNK}\\
\text{ASSERT}_\circ &\leftarrow \text{UNK}\\
\text{ASSERT}_\bullet &\leftarrow \text{UNK}
\end{aligned}
$$

Another example



$$\text{INIT}_\circ \leftarrow \text{UNK}$$
$$\text{INIT}_\bullet \leftarrow \text{ODD}$$
$$\text{LOOPHEAD}_\circ \leftarrow \text{ODD}$$
$$\text{LOOPHEAD}_\bullet \leftarrow \text{ODD}$$
$$\text{INCREMENT}_\circ \leftarrow \text{ODD}$$
$$\text{INCREMENT}_\bullet \leftarrow \text{EVEN}$$
$$\text{LOOPHEAD}_\circ \leftarrow \text{UNK}$$
$$\text{ASSERT}_\circ \leftarrow \text{EVEN}$$
$$\text{ASSERT}_\bullet \leftarrow \text{EVEN}$$
$$\text{LOOPHEAD}_\bullet \leftarrow \text{UNK}$$
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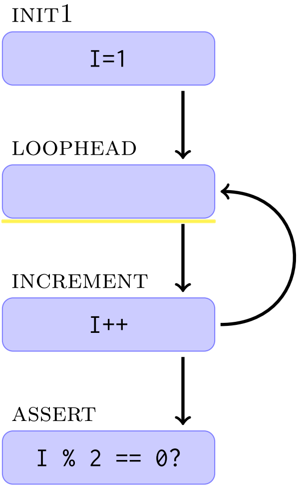
$$\text{ASSERT}_\bullet \leftarrow \text{UNK}$$

Another example



$\mathrm{INIT}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{INIT}_\bullet \leftarrow \mathrm{ODD}$

$\mathrm{LOOPHEAD}_\circ \leftarrow \mathrm{ODD}$

$\mathrm{LOOPHEAD}_\bullet \leftarrow \mathrm{ODD}$

$\mathrm{INCREMENT}_\circ \leftarrow \mathrm{ODD}$

$\mathrm{INCREMENT}_\bullet \leftarrow \mathrm{EVEN}$

$\mathrm{LOOPHEAD}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{ASSERT}_\circ \leftarrow \mathrm{EVEN}$

$\mathrm{ASSERT}_\bullet \leftarrow \mathrm{EVEN}$

$\mathrm{LOOPHEAD}_\bullet \leftarrow \mathrm{UNK}$

$\mathrm{INCREMENT}_\circ \leftarrow \mathrm{UNK}$

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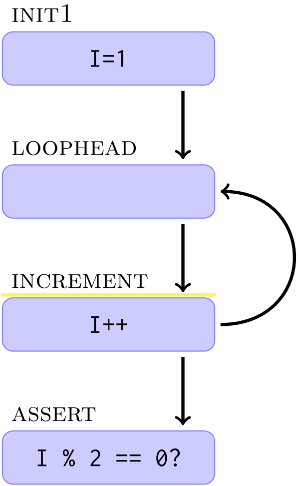
$\mathrm{ASSERT}_\circ \leftarrow \mathrm{UNK}$

$\mathrm{ASSERT}_\bullet \leftarrow \mathrm{UNK}$

Another example



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$\text{INIT}_\bullet \leftarrow \text{ODD}$

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$\text{INCREMENT}_\circ \leftarrow \text{ODD}$

$\text{INCREMENT}_\bullet \leftarrow \text{EVEN}$

$\text{LOOPHEAD}_\circ \leftarrow \text{UNK}$

$\text{ASSERT}_\circ \leftarrow \text{EVEN}$

$\text{ASSERT}_\bullet \leftarrow \text{EVEN}$

$\text{LOOPHEAD}_\bullet \leftarrow \text{UNK}$

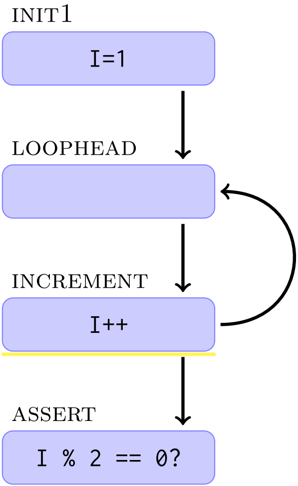
$\text{INCREMENT}_\circ \leftarrow \text{UNK}$

$\text{INCREMENT}_\bullet \leftarrow \text{UNK}$

$\text{ASSERT}_\circ \leftarrow \text{UNK}$

$\text{ASSERT}_\bullet \leftarrow \text{UNK}$

Another example



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\text{LOOPHEAD}_\circ &\leftarrow \text{UNK}\\
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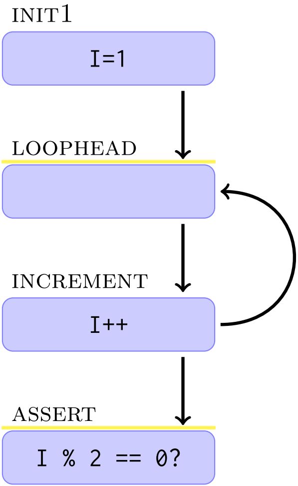
\end{aligned}
$$

Another example



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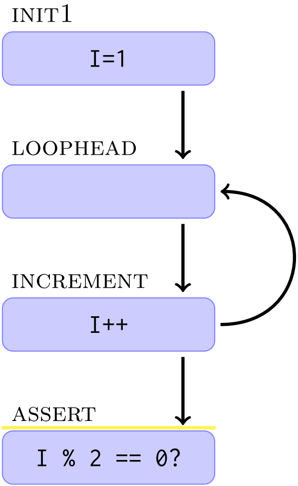
\end{aligned}$$

Another example



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$\text{LOOPHEAD}_\bullet \leftarrow \text{ODD}$

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$\text{INCREMENT}_\bullet \leftarrow \text{EVEN}$

$\text{LOOPHEAD}_\circ \leftarrow \text{UNK}$

$\text{ASSERT}_\circ \leftarrow \text{EVEN}$

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$\text{LOOPHEAD}_\bullet \leftarrow \text{UNK}$

$\text{INCREMENT}_\circ \leftarrow \text{UNK}$

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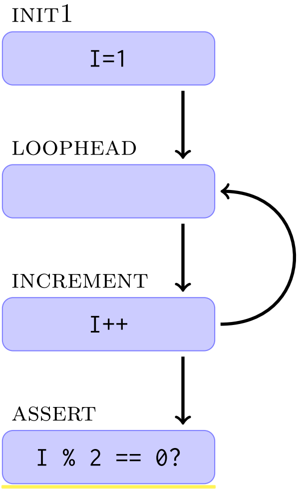

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\mathrm{INCREMENT}_\bullet &\leftarrow \mathrm{UNK}\\
\mathrm{ASSERT}_\circ &\leftarrow \mathrm{UNK}\\
\mathrm{ASSERT}_\bullet &\leftarrow \mathrm{UNK}
\end{aligned}
$$

Equation system

We can reframe the problem as a system of equations:

$$
\begin{aligned}
\mathrm{INIT}_{\circ} &= i \\
\mathrm{INIT}_{\bullet} &= f_{\mathrm{INIT}}(\mathrm{INIT}_{\circ}) \\
\mathrm{LOOPHEAD}_{\circ} &= \mathrm{INIT}_{\bullet} \sqcup \mathrm{INCREMENT}_{\bullet} \\
\mathrm{LOOPHEAD}_{\bullet} &= f_{\mathrm{LOOPHEAD}}(\mathrm{LOOPHEAD}_{\circ}) \\
\mathrm{INCREMENT}_{\circ} &= \mathrm{LOOPHEAD}_{\bullet} \\
\mathrm{INCREMENT}_{\bullet} &= f_{\mathrm{INCREMENT}}(\mathrm{INCREMENT}_{\circ}) \\
\mathrm{ASSERT}_{\circ} &= \mathrm{INCREMENT}_{\bullet} \\
\mathrm{ASSERT}_{\bullet} &= f_{\mathrm{ASSERT}}(\mathrm{ASSERT}_{\circ})
\end{aligned}
$$

Table of Contents

Introducing: Monotone Frameworks

A formal framework to define a data-flow analysis problem, which requires certain properties to hold for the defining components, and for which an algorithm with termination guarantees is available.

Building blocks

L the set of possible values we can associate to a label.

$\mathscr{F} = \{f : L \to L\}$ the *family* of transfer functions. Each function maps each $l \in L$ to $l' \in L$ in a different way. Later on, each label will be associated to one of such functions.

Constraints on L

L is known as the *property space*, and must respect the following constraints:

1. A relation $\sqsubseteq$ has to be defined: $\sqsubseteq$: $L \times L \rightarrow \{0,1\}$
2. $(L, \sqsubseteq)$ is a *partially ordered set*, i.e., $\sqsubseteq$ is
    1. reflexive
    2. anti-symmetric
    3. transitive

Ordering operator

I will call $\sqsubseteq$ *lower than or equal* operator
and $\sqsupset$ *greater than* operator

Graphical representation

A partially ordered set can be represented as a graph

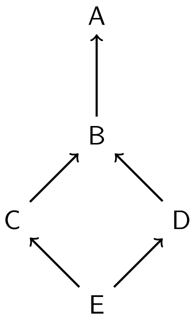
Nodes: elements in L

Edges: pairs of elements in $\sqsubseteq$: $L \times L$

Examples

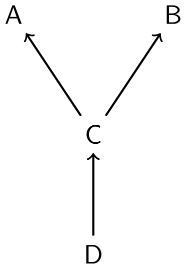
$$L = \{A, B, C, D, E\}$$

$$\sqsubseteq = \left\{ \begin{array}{c} (B, A), (C, B), (D, B), \\ (E, C), (E, D) \end{array} \right\}$$



$$L = \{A, B, C, D\}$$

$$\sqsubseteq = \{(C, A), (C, B), (D, C)\}$$



It's a partial ordering

Note that $(L, \sqsubseteq)$ is a *partially* ordered set: certain (l, l') might be non-comparable!

L has to be a *complete lattice*
but before getting into that, some definitions

Upper bound and least upper bound

Given $(L, \sqsubseteq)$ and $Y \subseteq L$

Upper bound of Y any element $l \in L$, such that $\forall l' \in Y, l' \sqsubseteq l$.

Least upper bound of Y ($\bigsqcup Y$) the only upper bound l of Y such that, for any other upper bound l_0 of Y, $l \sqsubseteq l_0$.

And, by duality:

Lower bound of Y any element $l \in L$, such that $\forall l' \in Y, l' \sqsupseteq l$.

Greatest lower bound of Y ($\bigsqcap Y$) the only lower bound l of Y such that, for any other lower bound l_0 of Y, $l \sqsupseteq l_0$.

Upper bound and least upper bound

Given $(L, \sqsubseteq)$ and $Y \subseteq L$

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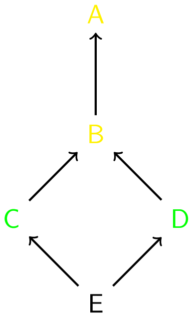
Upper bound

$$Y = \{C, D\}$$

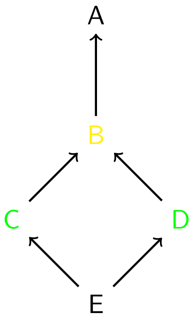
Upper bounds of $Y = \{A, B\}$



Least upper bound

$$Y = \{C, D\}$$

Least upper bound of $Y = \bigsqcup Y = \{B\}$

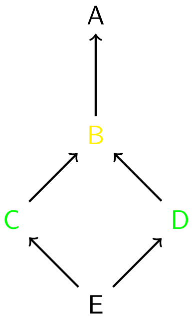


You can interpret $\bigsqcup$ as "the next common element upward"

Least upper bound

$$Y = \{C, D\}$$

Least upper bound of $Y = \bigsqcup Y = \{B\}$



You can interpret $\bigsqcup$ as "the next common element upward"

Complete lattice

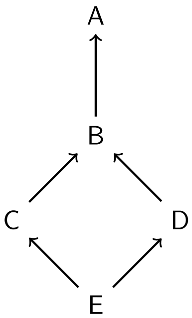
L is a complete lattice if each $Y \subseteq L$ has a least upper bound $\bigsqcup Y$ and greatest lower bound $\bigsqcap Y$.

We also define:

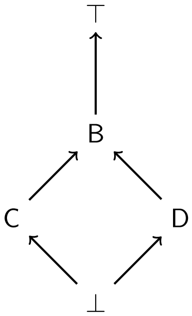
Top $\top = \bigsqcup L$

Bottom $\bot = \bigsqcap L$

Top and bottom



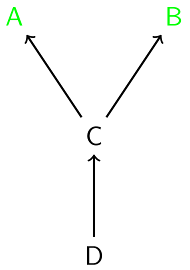
Top and bottom



Least upper bound

$$Y = \{A, B\}$$

Upper bound of $Y = ?$



Not a complete lattice!

Least upper bound

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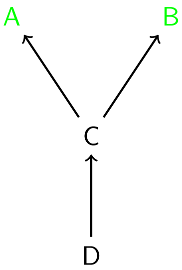


Not a complete lattice!

Ascending Chain Condition

Consider the facts that:

- L doesn't have to be finite.
- The relation $\sqsubseteq$ affects only certain pairs in $L \times L$.

The ACC states that:

All subsets Y of L where each pair of elements are comparable with $\sqsubseteq$, has to be finite.

Or, in symbols:

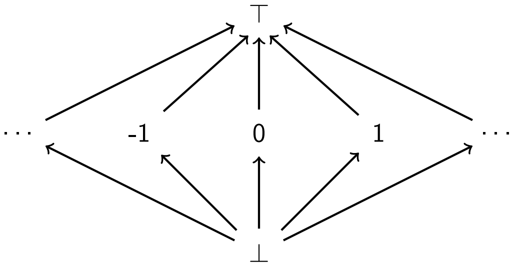
$$\forall Y \subseteq L : \forall l, l' \in Y, \left(l \sqsubseteq l'\right) \vee \left(l' \sqsubseteq l\right), |Y| < |\mathbb{N}|$$

Ascending Chain Condition (tl;dr)

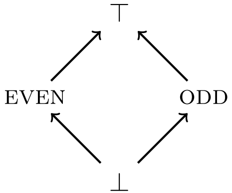
The graph of $(L, \sqsubseteq)$ must have a finite height h

Infinite L but finite h

The lattice of the constant propagation is infinite



The even-odd lattice

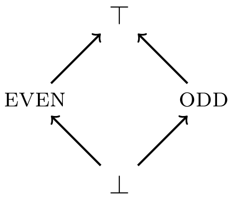


Interpretation:

- higher elements in the lattice are more *general*, they take into account more situations, they are *safer*, more *conservative*.
- lower elements in the lattice are more *specific*, *informative*, they carry a more useful information.

An analysis where everything is $\top$ is conservative but useless!

The even-odd lattice



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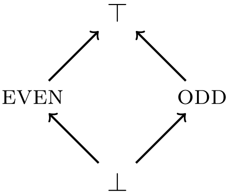

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Requirements on transfer functions

$\forall f \in \mathscr{F}$ we have that

1. f has to be montone: $l \sqsubseteq l' \Rightarrow f(l) \sqsubseteq f(l')$
2. $\forall f_\ell, f_\ell \in \mathscr{F}$
3. $\text{id} \in \mathscr{F}$
4. $\mathscr{F}$ is closed under function composition

Instances of Monotone Frameworks

$(L, \sqsubseteq, \bigsqcup, \mathcal{F})$	the Monotone Framework.
$\ell \in \mathrm{Lab}$	a label ℓ in the set of labels Lab.
$E \subseteq \mathrm{Lab}$	the set of extremal labels.
$F \subseteq (\mathrm{Lab} \times \mathrm{Lab})$	a pair of labels (ℓ, ℓ') in F representing an arc from ℓ to ℓ'.
$i \in L$	the extremal value i, i.e., the value initially associated to the extremal labels.
$\{f_\ell : \ell \in \mathrm{Lab}, f_\ell \in \mathcal{F}\}$	the set of transfer function associated to the label ℓ.

Direction

Analyses can also be *backward*!

- The extremal points are the return points of the function
- The flow F is reversed
- All the rest is the same!

tl;dr

To implement a Monotone Framework you need:

1. The graph of your function
2. Prepare a lattice
   - Make sure it's a complete lattice
   - Add an artificial $\bot$ if necessary
3. Elect one or more entry points and assign them a value
4. Define the transfer functions
   - What should happen on each label?
   - Ensure the transfer functions are monotone
5. Solve it!

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Let's formulate the equations

$$\begin{aligned}
\text{Analysis}_\circ(\ell) &= \begin{cases} i & \text{if } \ell \in E \\ \bigsqcup\{\text{Analysis}_\bullet(\ell') \mid (\ell',\ell) \in F\} & \text{otherwise} \end{cases} \\
\text{Analysis}_\bullet(\ell) &= f_\ell(\text{Analysis}_\circ(\ell))
\end{aligned}$$

Characterization

Analyses can be characterized by:

- $\bigsqcup$ as $\bigcap$ or $\bigcup$ (and $\sqcup$ is $\cup$ or $\cap$)
- F is either $flow(S_*)$ or $flow^R(S_*)$
- E is $\{init(S_*)\}$ or $final(S_*)$
- i specifies the initial or final analysis information
- f_ℓ is the transfer function for ℓ blocks

Characterization

- *forward analyses* have F to be $flow(S_*)$ and then $\text{Analysis}_\circ$ concerns entry conditions and $\text{Analysis}_\bullet$ concerns exit conditions. The equation system also supposes to have isolated entries.
- *backward analyses* have F to be $flow^R(S_*)$, and then $Analysis_\circ$ concerns exit conditions, and $Analysis_\bullet$ concerns entry conditions. The equation system also supposes to have isolated exits.

Characterization

- When $\bigsqcup$ is $\bigcap$ we require the *greatest* sets that solve the equations and we are able to detect properties satisfied by *all* the paths of execution reaching (or leaving) the entry (or exit) of a label; these analyses are often called *must analyses*.
- When $\bigsqcup$ is $\bigcup$ we require the *least* sets that solve the equations and we are able to detect properties satisfied by at *least one* execution path to (or from) the entry (the exit) of a label; these analyses are often called *may analyses*.

Characterization

Given the properties seen before, we can characterize a data flow analysis with a triple:

$$\{\bigcap, \bigcup\} \times \{\rightarrow, \leftarrow\} \times \{\uparrow, \downarrow\} \text{ where:}$$

- $\rightarrow$ means *forwards*, $\leftarrow$ means backwards
- $\downarrow$ means smallest and $\uparrow$ means largest
- This should lead to *eight* types of analyses, but given that we associate $\bigcap$ with $\uparrow$ and $\bigcup$ with $\downarrow$ the cube collapse to a square
- In the end we usually have analyses of the followinf four types: $(\bigcap, \rightarrow, \uparrow)$, $(\bigcup, \rightarrow, \downarrow)$, $(\bigcap, \leftarrow, \uparrow)$, $(\bigcup, \leftarrow, \downarrow)$

Available expressions analysis

Elem	Value
L	$P(AExp_*)$
$\sqsubseteq$	$\supseteq$
$\bigsqcup$	$\bigcap$
$\bot$	$AExp_*$
ι	$\emptyset$
E	$\{init(S_*)\}$
F	$flow(S_*)$

Reaching definitions analysis

Elem	Value
L	$P\left(Var_* \times Lab_*^?\right)$
$\sqsubseteq$	$\subseteq$
$\bigsqcup$	$\bigcup$
$\bot$	$\emptyset$
ι	$\{(x,?) \mid x \in FV(S_*)\}$
E	$\{init(S_*)\}$
F	$flow(S_*)$

Very busy expressions analysis

Elem	Value
L	$P(AExp_*)$
$\sqsubseteq$	$\supseteq$
$\bigsqcup$	$\bigcap$
$\bot$	$AExp_*$
ι	$\emptyset$
E	$final(S_*)$
F	$flow^R(S_*)$

Live variables analysis

Elem	Value
L	$P(Var_*)$
$\sqsubseteq$	$\subseteq$
$\bigsqcup$	$\bigcup$
$\bot$	$\emptyset$
ι	$\emptyset$
E	$final(S_*)$
F	$flow^R(S_*)$

Maximum Fixed Point

The MFP is a way to solve the system of equations:

- It's scalable
- Its termination is guaranteed
- Offers good solutions

Initialization

Data: A Monotone Framework $(L, \mathscr{F}, F, E, i, f)$
Result: $MFP_\circ$, $MFP_\bullet$

$Worklist = \{\}$;
$tmp = \{\}$;
foreach $(\ell, \ell') \in F$ **do**
  $Worklist.enqueue((\ell, \ell'))$;
end
foreach $\ell \in E$ **do**
  $tmp[\ell] = i$;
end
foreach $\ell \in F \backslash E$ **do**
  $tmp[\ell] = \bot$;
end

Iterative refinement

```
while not Worklist.empty() do
    (ℓ, ℓ') = Worklist.pop();
    r = f_ℓ(tmp[ℓ]);
    if r ⋢ tmp[ℓ'] then
        tmp[ℓ'] = tmp[ℓ'] ⊔ r;
        foreach ℓ'' such that ∃(ℓ', ℓ'') ∈ F do
            Worklist.enqueue((ℓ', ℓ''));
        end
    end
end
```

Finalization

foreach $\ell \in F$ **do**

$MFP_\circ[\ell] = tmp[\ell]$;

$MFP_\bullet[\ell] = f_\ell(tmp[\ell])$;

end

An interpretation

Start from the lowest, incorrect, solution and *inflate* it until it becomes correct

Complexity upper bound

$$O\left(e \cdot h\right)$$

e number of edges in the CFG

h height of the lattice L

The MOP solution

Given the instance of a Monotone Framework, we can formulate the problem differently

- Consider each label
- For each label, compute all the paths from E
- Compute the transfer function for that path
- Apply the transfer function to i
- Merge all the results using $\bigsqcup$

What is a path?

A path is a finite sequence of labels from $\ell_1 \in E$ to ℓ,
either excluding $path_\circ(\ell)$ or including it $path_\bullet(\ell)$

$$path_\circ(\ell) = \left\{ [\ell_1, \ldots, \ell_{n-1}] : \begin{cases} n \geq 1 \\ \forall i < n : (\ell_i, \ell_{i+1}) \in F \\ \ell_n = \ell \\ \ell_1 \in E \end{cases} \right\}$$

$$path_\bullet(\ell) = \left\{ [\ell_1, \ldots, \ell_n] : \begin{cases} n \geq 1 \\ \forall i < n : (\ell_i, \ell_{i+1}) \in F \\ \ell_n = \ell \\ \ell_1 \in E \end{cases} \right\}$$

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Transfer function of a path

Simply combine all the transfer functions

$$\vec{\ell} = [\ell_1, \ldots, \ell_n]$$

$$f_{\vec{\ell}}(l) = f_{\ell_n}\left(f_{\ldots}\left(f_{l_2}\left(f_{\ell_1}(l)\right)\right)\right) = f_{\ell_n} \circ \cdots \circ f_{\ell_1}$$

The MOP equations

$$MOP_\circ(\ell) = \bigsqcup \left\{ f_{\vec{\ell}}(\iota) \mid \vec{\ell} \in path_\circ(\ell) \right\}$$

$$MOP_\bullet(\ell) = \bigsqcup \left\{ f_{\vec{\ell}}(\iota) \mid \vec{\ell} \in path_\bullet(\ell) \right\}$$

In general, MOP is undecidable.
It might not terminate

Relation between MFP and MOP

It can be proven that

$$MOP \sqsubseteq MFP$$

Relation between MFP and MOP

It can be proven that

$$MOP = MFP$$

if the transfer functions are distributive, i.e.,

$$f_\ell(l_1 \sqcup l_2) = f_\ell(l_1) \sqcup f_\ell(l_2)$$

$MFP \neq MOP$ example

The constant propagation analysis is not distributive

$$
\begin{aligned}
a \sqcup a &= a \\
a \sqcup b &= \top
\end{aligned}
$$

$$
\begin{aligned}
\ell : a &= x^2 \\
f_\ell(1) &= 1 \\
f_\ell(-1) &= 1 \\
f_\ell(1 \sqcup -1) &= f(\top) = \top \\
f_\ell(1) \sqcup f_\ell(-1) &= 1 \sqcup 1 = 1
\end{aligned}
$$

$MFP \neq MOP$ example

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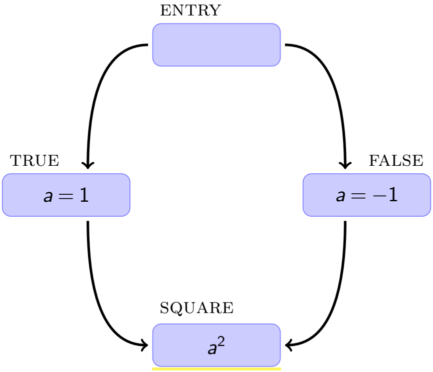
MFP



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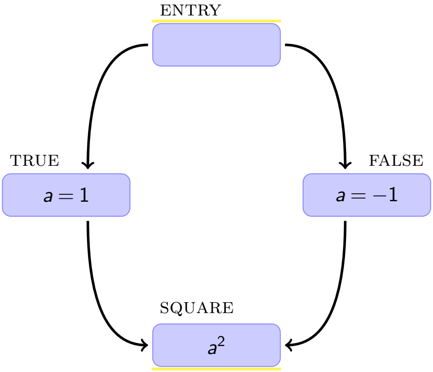
MFP



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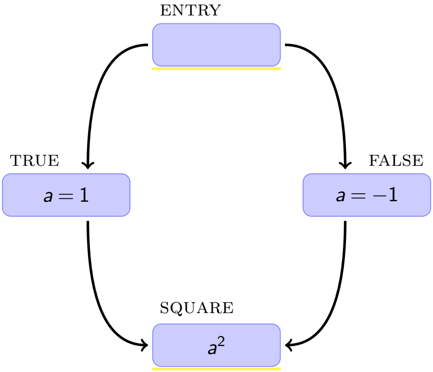

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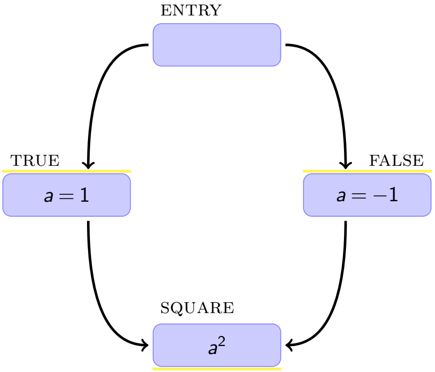

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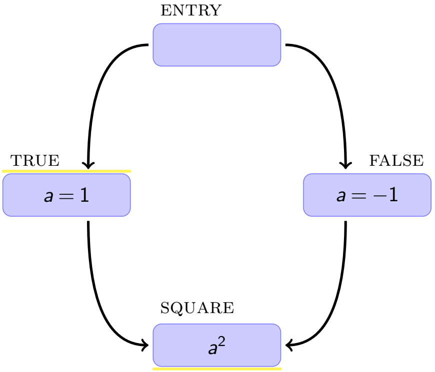

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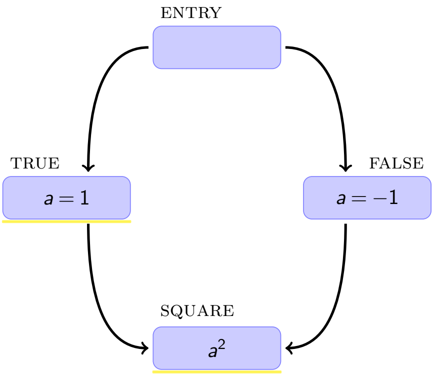

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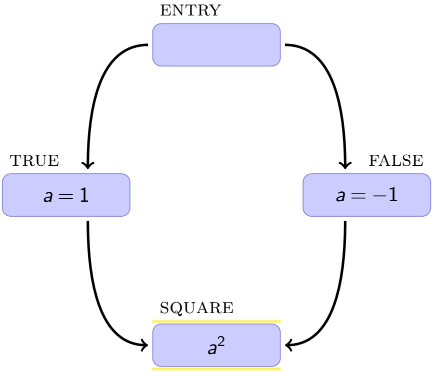

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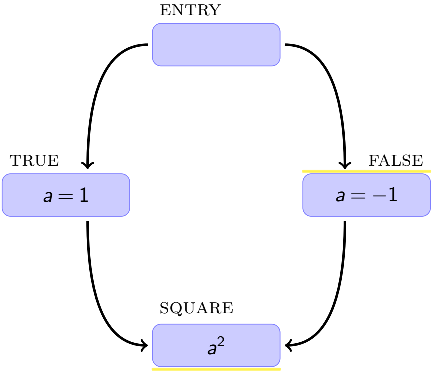

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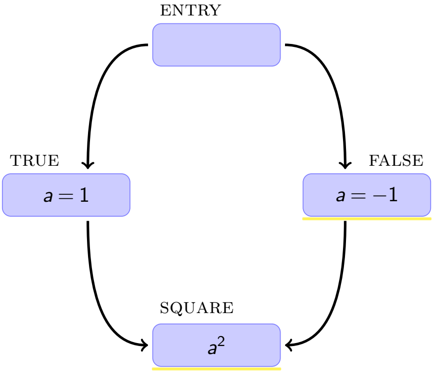

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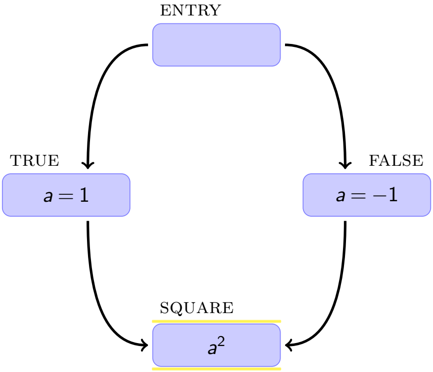

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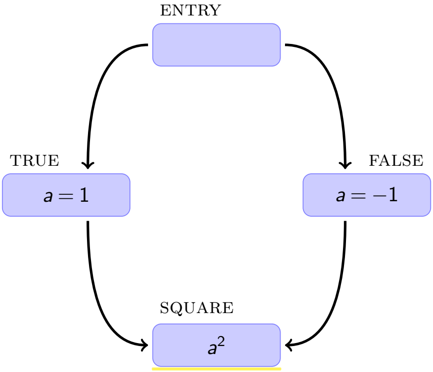

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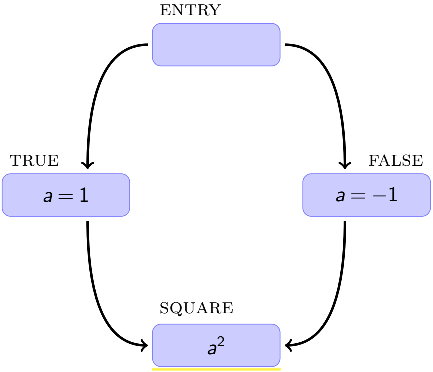
MOP



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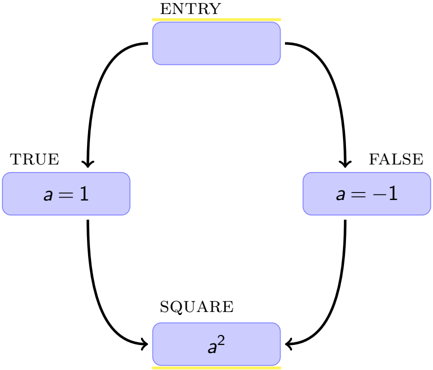

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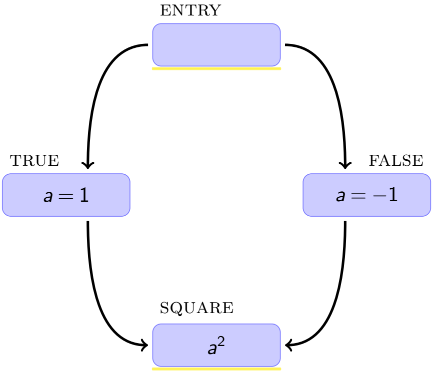

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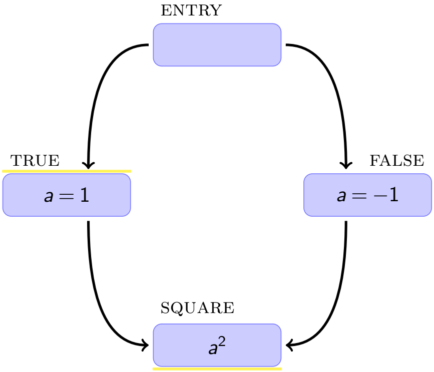

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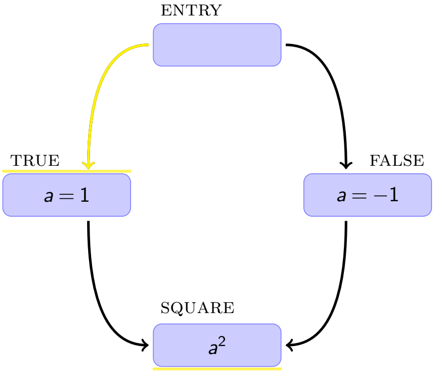

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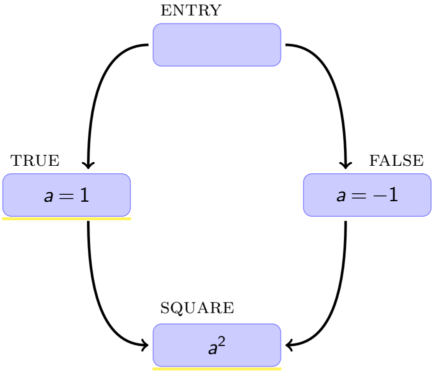

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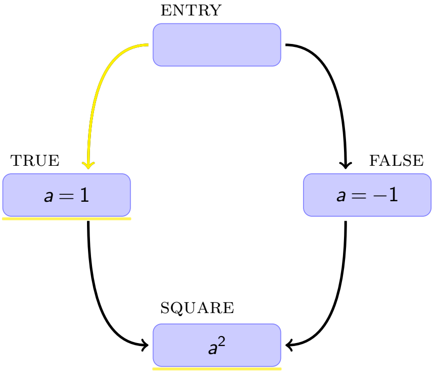

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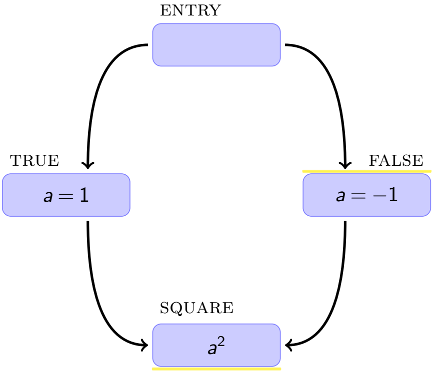

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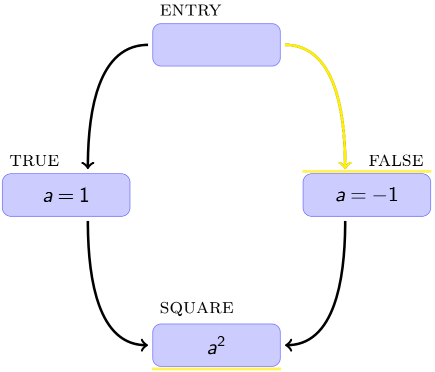

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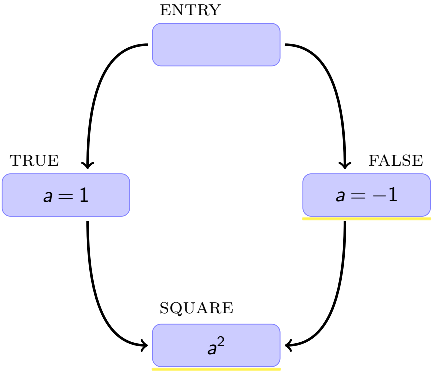

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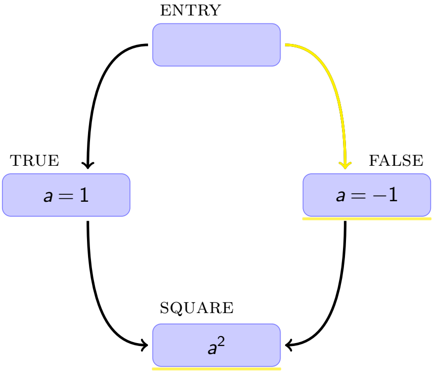

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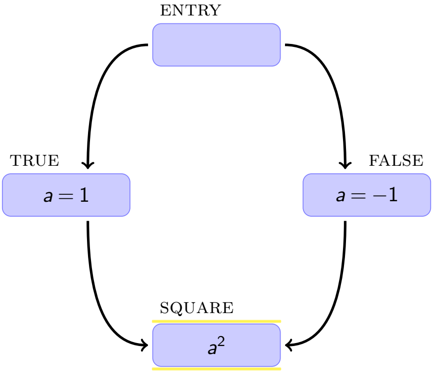

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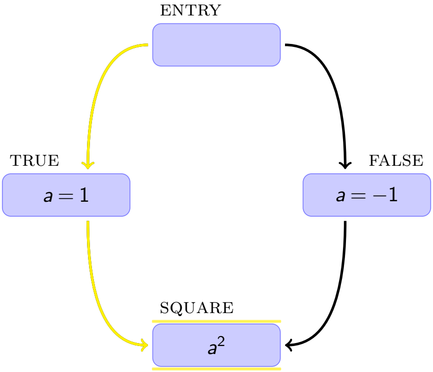

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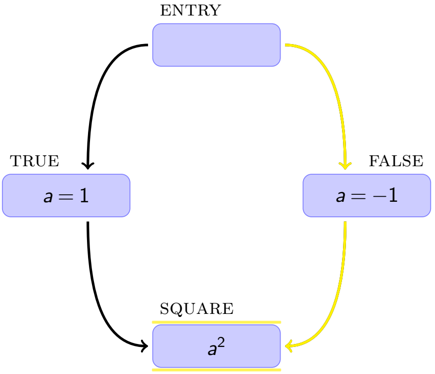

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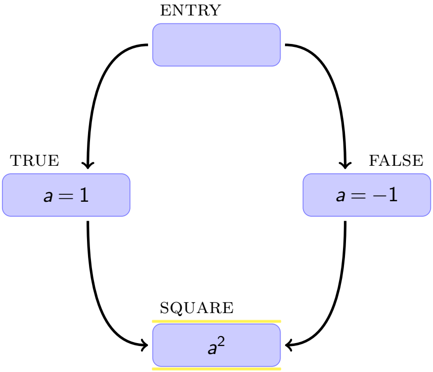

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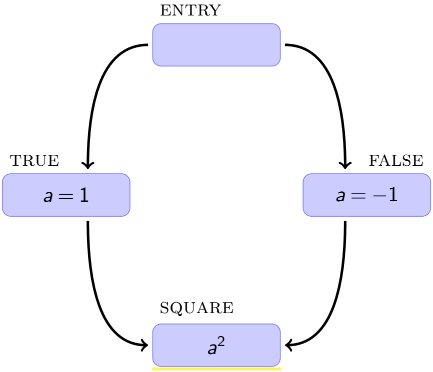

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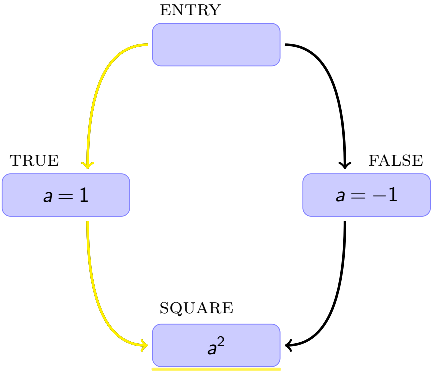

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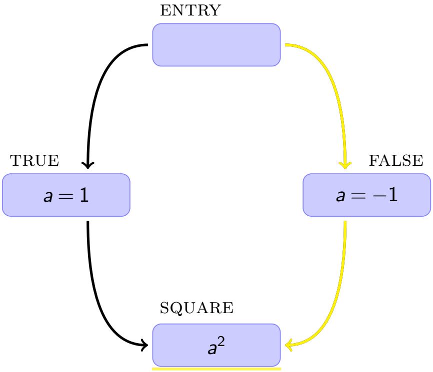

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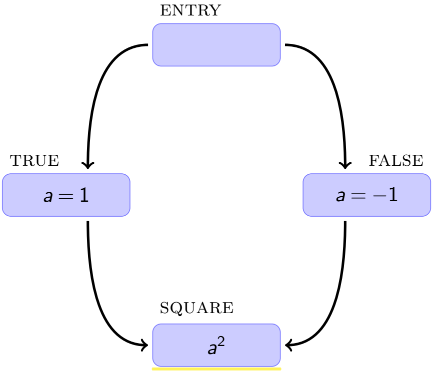
\text{ENTRY}_{\bullet} &\leftarrow \top \\
\text{TRUE}_{\circ} &\leftarrow \top \\
\text{TRUE}_{\bullet} &\leftarrow 1 \\
\text{FALSE}_{\circ} &\leftarrow \top \\
\text{FALSE}_{\bullet} &\leftarrow -1 \\
\text{SQUARE}_{\circ} &\leftarrow 1 \sqcup -1 = \top \\
\text{SQUARE}_{\bullet} &\leftarrow 1 \sqcup 1 = 1
\end{aligned}
$$

MOP



$$
\begin{aligned}
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\text{ENTRY}_\bullet &\leftarrow \top\\
\text{TRUE}_\circ &\leftarrow \top\\
\text{TRUE}_\bullet &\leftarrow 1\\
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\text{SQUARE}_\bullet &\leftarrow 1 \sqcup 1 = 1
\end{aligned}
$$

Table of Contents

Divide et impera

The key for every analysis it KISS:

- Divide the analysis part from the transformation part
- An analysis should do a single thing
- Dividing the tasks simplifies also the debug effort

Lattice design

- Spend some time in the design phase on the lattice, it will save time later
- If you work with set of things, the lattice will be obvious

Transfer function

- Remember that we have available a set of transfer functions
- Check the monotonicity of each transfer function

A word of advice

When trying to design an analysis:

- keep it simple, make it do only one thing
- work with set of things, the lattice will be obvious
- check the monotonicity of each transfer function

Visit order for the MFP

- MFP algorithm explores the graph depth-first
- This is suboptimal, you should always try to visit first the nodes whose change might affect the most nodes.
- For instance, the entry node!
- Rule: sort the node list in reverse-post order, and, among the nodes that need to be visited, always take the first appearing in the list.

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References I

Flemming Nielson, Hanne Riis Nielson, and Chris Hankin. *Principles of Program Analysis*. Jan. 1999. DOI: 10.1007/978-3-662-03811-6.

Credits

- The slides used in this seminar have been created by Alessandro Di Federico. I want to thank him for letting me use the material.

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